

# Stochastic Star Identification

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**An approach to star identification based on comparing observed pattern statistics with the precomputed star cataloged statistics is suggested. The identification criterion is based on evaluating a posteriori probabilities of designated star sequences obtained from observing different star fields. Numerical results based on a specific algorithm are presented. A number of references for other approaches are cited.**

## Introduction

**O**PERATION and performance of spacecraft control systems will be significantly improved with the ability to autonomously and precisely calculate spacecraft attitude by means of star sensors. With recent advancements in charge-coupled device (CCD) star trackers<sup>1–4</sup> and NASA's emphasis on developing cost-effective spacecraft, a star tracker with full-sky recognition based on multiple star observations will have a significant impact in terms of cost, limitations, and operational complexity of spacecraft and space-based instruments. The ability to identify guide stars autonomously will provide more efficient, robust, and independent modes of attitude determination and fault or loss-of-attitude recovery.

Once the star field is correctly identified, spacecraft attitude can be determined by generating the direction cosine matrix from one pair of identified stars or solving for the attitude with the least-squares method using all the identified stars in the field of view (FOV)<sup>5</sup> or various other methods. Having established the initial attitude, subsequent star pattern recognition and attitude determination processing will be greatly simplified (tracking known stars becomes an issue here).

Although many algorithms have been proposed for star pattern identification, significant limitations in terms of parametric sensitivities, flexibility, and implementability have not yet been overcome. For example, angular separation techniques that rely on matching all possible pairs of observed stars to cataloged star pairs<sup>6</sup> carry a tremendous computational and memory burden due to  $N(N-1)/2$  combinations (although this number can be reduced by considering only pairs that do not exceed the camera FOV, a number of candidate pairs is still quite substantial for moderate FOV and star magnitude range). In addition, the problem of accommodating such uncertainties as the failure to match a few of the connected pairs in the observed star group is not trivial and often leads to implementation of many ad hoc and cumbersome selection/elimination criteria that can cause serious flaws.<sup>7</sup> Grouping star pairs in terms of triangles<sup>8,9</sup>—this implies  $N(N-1)(N-2)/6$  combinations—has the same problem as the matching of all possible pairs. Although restricting the triangle pattern to that of only nearby stars<sup>10</sup> or setting a tree structure of nearby stars pairing with a root star<sup>11</sup> may alleviate the computation and memory problem, the sensitivity to distortions and missing or phantom stars becomes more distinct. The advantage of the triangle match, however, is that triangle patterns are more distinct than pairs, thus eliminating many spurious data points, and handling triangle groups is much more manageable than generic polygonal groups. In addition, positive identification may still be achieved even when the algorithm fails to match a few of the many possible triangles.

An alternative to the pairwise comparison is some sort of template matching between the observed star field and a candidate star field from a star catalog projected onto the camera focal plane. For example, by providing an initial guess of the a priori attitude by matching a confident star pair in the observed field with catalog pairs or using other means, a nonlinear least-squares technique can then be employed to confirm the identification matches of all the stars based on minimum mean-square distance errors between observed and cataloged pairs (one-to-one pairing) and fine-tune the estimated attitude.<sup>12,13</sup> Although this technique may be more desirable in terms of robustness, by providing less sensitivity to position errors, phantom stars, and missing stars, the methodology of deriving initial guesses of the a priori attitude to achieve autonomous, all-sky star identification may encompass and inherit all the difficulties associated with the above angular separation approach, in addition to the computational burden and convergence problems associated with the iterative nonlinear least-squares technique.

Another pattern-match approach in this template-matching class is to analyze the error vectors that join observed stars to their cataloged counterparts transformed to zero attitude.<sup>14</sup> Using the assumption that the basic star pattern is conserved in the presence of distortions and observation noise, these error vectors should all be parallel to one another and have the same magnitude. This approach may work well in an ideal case (i.e., a noise-free environment), but real observation data, as shown in DeAntonio et al.,<sup>7</sup> may possess significant distortions (especially in the parallelism of the error vectors) that are sufficient to cause confusion in terms of the correct star-pairing assignments between observed and cataloged stars (especially when there are more cataloged stars being compared with observed stars).

Identification techniques based on neural network and fuzzy logic have also been suggested for star pattern recognition,<sup>15–17</sup> and other types of feature extraction techniques<sup>18–21</sup> tend to be more appropriate for ground processing of photographic plates rather than on-board processing to establish spacecraft attitude. Although the concept of using stars to establish spacecraft attitude has been discussed for over 30 years,<sup>22</sup> in the authors' opinion, an optimal solution to the star identification problem that can be applied to a wide range of observation fields (large and small FOV, bright and dim stars, small and large perturbations to observed fields, and with or without a priori attitude knowledge) has not been formulated. It appears that, in terms of extracting the necessary information from a single observation field to match observed stars with cataloged stars, there are not many avenues left to explore, and the above-mentioned difficulties may be characteristic of the way the star identification problem is addressed (i.e., obtaining the solution from a single observation).

On the other hand, if the problem is posed in a stochastic way (i.e., given a sequence of observations and viewing motion up to time  $k$ , identify the currently observed star pattern), a rich foundation of optimal filtering and estimation theory<sup>23</sup> can be applied to solve it. The major assumption here is that the spacecraft is permitted to turn and acquire different observations of the sky to establish its attitude, where the relative attitude of one scene to the next is known. This assumption is quite reasonable, since, in practice, such a maneuver would be required when the observed star pattern does

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not yield positive identification (such an approach is the baseline for Cassini spacecraft<sup>24</sup> attitude initialization). The approach of using star scanners for spin-stabilized spacecraft is another example of establishing attitude after scanning through a significant portion of the sky.

In this paper, we formulate a stochastic star identification technique based on multiple observations. The stochastic process is defined by a sequence of vectors representing just the statistics of star-clustering features associated with a designated star (e.g., star density, maximum/average/range/standard deviation of star magnitude and angular separation, etc.) at different observation fields. At each observation point, the set of measured statistics is compared with the precomputed statistics for each cataloged star, and the probability of each cataloged star being corrected given the history of observed statistics up to time  $k$  is then updated. Knowledge of the turn direction and offset distance between observation frames is required. Adequate number of observed stars in the FOV is also required in order to generate good statistics. However, the main advantage of this approach is that, at any particular time, we have a system to assess the likelihood of each cataloged star to the designated star in the observed field. Furthermore, since comparisons are based on a set of statistics instead of individual pair metrics, computation and memory requirements grow linearly with the number of stars in the catalog instead of the above-mentioned pairwise combinations, and a larger degree of position and magnitude distortions can be tolerated.

The first part of this paper describes the mathematical formulation of the proposed stochastic star identification technique. The second part discusses a specific algorithm and provides numerical results of this approach.

### Methodology

Let  $f$  be a mapping function between the inertial spacecraft boresight  $b$  and cataloged star  $s$ , i.e.,

$$f: b_k \longrightarrow s_k; b_k \in \mathbf{R}^3 \times [-\pi, \pi]$$

and

$$s_k \in C \equiv \text{index set of catalog stars}$$

where  $b_k$  represents the boresight location and orientation (a unit vector with a rotation about that vector) at time  $k$  and  $s_k$  represents the cataloged star (which belongs to a discrete set of star indices) associated with  $b_k$ . Thus, for any given boresight, there is a deterministic rule to designate the corresponding star within a given FOV. An example of  $f$  would be the selection of the closest star to the boresight location. Different types of mappings that incorporate both the distance and star magnitude constraints or other variables are also feasible. Note that this mapping may not be invertible; i.e., one particular star may correspond to a range of boresight values within some neighborhood (region) around the star. However, as long as this mapping satisfies some continuity separation constraint (i.e., a particular star may not map to different boresight values that belong to different regions), this ambiguity will not affect the identification and attitude determination results.

Let  $g$  be a mapping function between the cataloged star and its associated clustering statistics  $x$ , which characterize the star pattern surrounding each cataloged star for a given FOV,

$$g: s_k \longrightarrow x_k; x_k \in \mathbf{R}^m$$

This mapping should be a function of the camera FOV and must provide statistical measures to characterize the neighboring stars. A variety of metrics corresponding to star patterns and clustering features can be applied here. A simple example of  $x$  would be star density, average magnitude, and the standard deviation of the magnitude. Therefore, for a given star field with a known designated star, a vector comprised of representative statistics with respect to its neighboring stars can be determined.

Furthermore,  $x_k$  can be described as a random vector since each observation field is perturbed by some random variables that distort the true star locations and magnitudes. Thus, given a sequence of measured  $\{x_0, x_1, \dots, x_k\}$  (denoted as  $X^k$ ), the probability that

a corresponding set of stars  $\{s_0, s_1, \dots, s_k\}$  (denoted as  $S^k$ ) is a true set possesses the following relationship, according to Bayes's rule:

$$\begin{aligned} p(S^k | X^k) &= \frac{p(S^k, X^k)}{p(X^k)} = \frac{p(s_k, S^{k-1}, x_k, X^{k-1})}{p(x_k, X^{k-1})} \\ &= \frac{p(x_k, s_k | X^{k-1}, S^{k-1}) p(S^{k-1} | X^{k-1}) p(X^{k-1})}{p(x_k | X^{k-1}) p(X^{k-1})} \\ &= \frac{p(x_k | X^{k-1}, S^{k-1}, s_k) p(s_k | X^{k-1}, S^{k-1}) p(S^{k-1} | X^{k-1})}{\sum_{r_k \in C} p(x_k | X^{k-1}, R^{k-1}, r_k) p(r_k | X^{k-1}, R^{k-1}) p(R^{k-1} | X^{k-1})} \end{aligned} \quad (1)$$

Note that all possible star sets corresponding to  $\{x_0, x_1, \dots, x_k\}$  that will be assigned probability values are represented in  $R^k$ . The above equation is quite complicated and requires definitions of conditional distribution functions that may be difficult to derive. To simplify this further, assume that the statistics  $x_i$  associated with each  $s_i$  are independent of  $x_j$  and  $s_j$  for  $i \neq j$ . This assumption may not be quite accurate, since neighboring locations of star fields may indeed depend on each other if the observed star fields overlap each other for time  $i$  and time  $j$ . However, as we will show in the next section, that convergence to 1 for the probability of the true set can still be achieved without having an accurate account of the conditional distribution functions. Thus, the a posteriori probabilities for each designated star sequence at time  $k$  can be given by

$$p(S^k | X^k) = \frac{p(x_k | s_k) p(s_k) p(S^{k-1} | X^{k-1})}{\sum_{r_k \in C} p(x_k | r_k) p(r_k) p(R^{k-1} | X^{k-1})} \quad (2)$$

This result is obvious. If each star-field observation is independent, then the probability of a sequence of events is simply the multiplication of the probability of each individual event. This recursive expression implies that only the previous values of  $p(S^k | X^k)$  are required for propagating the a posteriori probabilities. The denominator is just a normalizing constant to obtain the correct probability range. In addition, the term  $p(r_k)$  provides a systematic way to initially bias each star field if there is some a priori knowledge about the spacecraft pointing location in a probabilistic sense. Given that there is no a priori knowledge about the initial pointing direction and each star in the catalog has equal probability of being observed,

$$p(S^k | X^k) = \frac{p(x_k | s_k) p(S^{k-1} | X^{k-1})}{\sum_{r_k \in C} p(x_k | r_k) p(R^{k-1} | X^{k-1})} \quad (3)$$

Therefore, the only term required to propagate the probability of a sequence of stars being observed up to time  $k$  is the individual conditional density function  $p(x_k | s_k)$ .

In the next section, we will describe the algorithm in detail and results based on a specific set of  $x_k$  and the assumption on  $p(x_k | s_k)$ .

### Numerical Examples

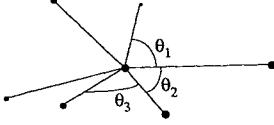
#### Representative Statistics for Star Field

Given a star field with the designated star  $s_k$  ( $s_k$  is defined by the boresight location according to the rule  $f$ ; e.g.,  $f$  is the mapping that selects the nearest star closest to the boresight), there are no specific guidelines to define the statistics  $x_k$  that adequately represent the star pattern. Using an ad hoc approach followed by trial and error, one may find an efficient set of statistics that is applicable to a broad range of conditions (different FOV and magnitude sensitivity). Here, we present an example of  $x_k$  that appears to perform reasonably well during our simulation. Define  $x$  as

$$x = \begin{pmatrix} n \\ \mu_m \\ \sigma_m \\ \mu_d \\ \sigma_d \\ \sigma_\theta \end{pmatrix} \quad (4)$$

**Table 1** Calculation of the a posteriori probabilities

$p(s)$	$s_0$	$p(x_0   s_0)$	$\Delta_0$	$p(S^0   X^0)$	$s_1$	$p(x_1   s_1)$	$\Delta_1$	$p(S^1   X^1)$	...
$1/N$	1	$p_{10}$	$\Delta_{10} = p_{10}/N$	$\rho_{10} = \Delta_{10}/\Sigma_0$	$s_{11}$	$p_{11}$	$\Delta_{11} = p_{11}\rho_{10}$	$\rho_{11} = \Delta_{11}/\Sigma_1$	...
$1/N$	2	$p_{20}$	$\Delta_{20} = p_{20}/N$	$\rho_{20} = \Delta_{20}/\Sigma_0$	$s_{21}$	$p_{21}$	$\Delta_{21} = p_{21}\rho_{20}$	$\rho_{21} = \Delta_{21}/\Sigma_1$	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$1/N$	$N$	$p_{N0}$	$\Delta_{N0} = p_{N0}/N$	$\rho_{N0} = \Delta_{N0}/\Sigma_0$	$s_{N1}$	$p_{N1}$	$\Delta_{N1} = p_{N1}\rho_{N0}$	$\rho_{N1} = \Delta_{N1}/\Sigma_1$	...
$\Sigma_0 = \Delta_{10} + \Delta_{20} + \dots + \Delta_{N0}$				$\Sigma_1 = \Delta_{11} + \Delta_{21} + \dots + \Delta_{N1}$					

**Fig. 1** Neighboring angle definition of star pattern.

where  $n$  is the number of measured stars in the FOV,  $\mu_m$  the average magnitude,  $\sigma_m$  the standard deviation of observed magnitudes,  $\mu_d$  the average angular distance to the designated star,  $\sigma_d$  the standard deviation of the angular distance to the designated star, and  $\sigma_\theta$  the standard deviation of the neighboring angles ( $\theta$ ) formed by lines radiating from the designated star (see Fig. 1 for the description of  $\theta$ ).

As mentioned earlier, there are different ways to define the vector  $x$ . For example, the maximum magnitude or the range value could have been used instead of the average value. A different description for describing the clustering of neighboring stars based on some geometric moments of the polygon or ellipse that define the star cluster boundary can also be selected instead of  $\theta$ . However, the main focus of this paper is to describe the implementation and demonstrate the proposed methodology for star identification, not what statistics to use. We therefore will leave this issue to future papers.

Finally, we must describe the conditional density function  $p(x_k | s_k)$ . Again, this term can be quite complex since each element of the vector  $x$  is interdependent and possesses different distributions. Although one can exercise the Monte Carlo simulation to accurately estimate this function, for simplicity, we will just approximate  $p(x_k | s_k)$  by the Gaussian distribution (this assumption works reasonably well during the simulation). Thus,

$$p(x_k | s_k) \approx (2\pi)^{-m/2} |Q^{-1}|^{1/2} \times \exp\left[-\frac{1}{2}(x_{k|s_k} - \xi_{k|s_k})^T Q^{-1}(x_{k|s_k} - \xi_{k|s_k})\right] \quad (5)$$

where  $\xi_{k|s_k}$  represents the true mean of the random vector  $x_{k|s_k}$  and  $Q$  is the covariance matrix assumed constant for all  $k$  and  $s$  values. Note that  $\xi$  for each star  $s$  can be estimated off-line using the data (star coordinates and magnitudes) provided by the selected guide star catalog. However, the value for the covariance  $Q$  will have to be predicted based on sensing instrument characteristics and measurement noise.

#### Algorithm

Having defined the procedure to generate the statistics for a given observation field, the implementation of the proposed methodology can easily be described as shown in Table 1.

At time 0, all the stars in the catalog are given equal priors ( $1/N$ ), where  $N$  is the number of stars in the catalog. By comparing the measured statistics  $x_0$  with the catalog statistics  $\xi_0$ ,  $p(x_0 | s_0)$  can be computed for each catalog star. The a posteriori probabilities based on the first observation  $p(S^0 | X^0)$  are then calculated by multiplying  $p(x_0 | s_0)$  by the priors with the appropriate normalizing factor.

At time 1, the spacecraft may assume a different pointing location, and each star in the catalog set at  $s_1$  must be rearranged to match the stars in the previous star set  $s_0$  in terms of the expected observation of the newly designated star caused by the known offset and turning direction with respect to the unknown initial pointing attitude. As shown in the  $s_1$  column of Table 1, elements in  $s_1$  still belong to the catalog index set  $C$ , but the order is reshuffled (including its pre-

computed statistics, which then becomes  $\xi_1$ ) to match the previous star set caused by the known turn. This is the most time-consuming step, since the coordinates of each star in  $s_0$  must be shifted by the known offset value and given each new boresight position, and the newly designated star (defined by  $f$ ) is then correctly registered in  $s_1$ . Some sort of optimization can be performed here if the turning of the spacecraft is preplanned, e.g., capture a new frame every 10 deg turn about the sun line. Thus, candidate stars in the 10 deg distance can be precomputed for each star. Once all stars are placed in the correct order in  $s_1$ , the same process to update the a posteriori probabilities  $p(S^1 | X^1)$  is then repeated with  $p(x_0 | s_0)$  assuming the role of the priors.

Having described the first two steps of the algorithm, the processing of subsequent frames can be accomplished in the same way. Once the designated star has been identified (i.e.,  $\rho_{ik} \rightarrow 1$  and  $\rho_{jk} \rightarrow 0, \forall j \neq i$ ), matching neighboring stars to provide at least two reference vectors for attitude determination becomes trivial. An alternative to simultaneously provide two star references is to divide the camera FOV into two fields and run the above algorithm in parallel.

#### Test Setup

To assess the feasibility of the proposed stochastic star identification methodology, the test software employing a subset of the Yale Bright Star Catalog (containing 918 stars, to magnitude 4.5) to simulate the sky was developed. Figure 2 depicts the functional flow of the test software.

Coordinates and magnitudes of the star field can be perturbed by white Gaussian noise using the prespecified noise variances to create a nonideal measurement environment. The boresight location, FOV, turn increment, and direction are also programmable.

#### Simulation Results

A sample of how the a posteriori probabilities converge to the correct designated star is described in Table 2.

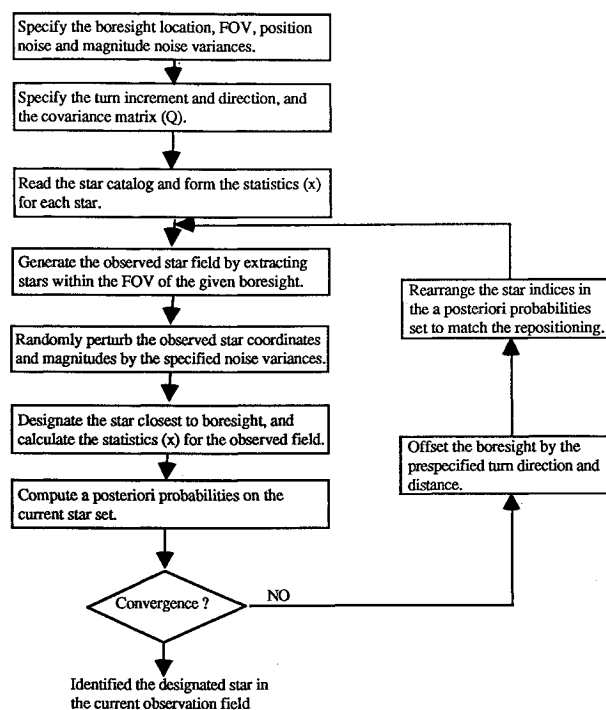
The convergence is quite rapid (less than five steps) for the given boresight location at star 13 (Betelgeus in the Orion constellation) in the Yale catalog with the 30 deg FOV and 10 deg turn along the right ascension axis. To represent measurement error, the  $1\sigma$  perturbation of 1 deg in right ascension and declination and 0.6 magnitude perturbation are employed in the simulation of observed star fields. The covariance matrix  $Q$  for the Gaussian density function  $p(x_k | s_k)$  is approximated by

$$Q = \begin{bmatrix} 4(\text{star}^2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.36(\text{mag}^2) & 0 & 0 & 0 & 0 \\ 0 & 0 & .36(\text{mag}^2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(\text{deg}^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(\text{deg}^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01(\text{deg}^2) \end{bmatrix}$$

The robustness performance of this algorithm is also assessed by analyzing the outcome after 100 test runs using different observation noise sequences. The average a posteriori probability of the correct star after five steps is 0.976, and the values range from 0.709 to 0.999.

**Table 2** Convergence example of a posteriori probability

Time	Number of stars with probability > 1/918	Posteriori probability of correct star
0	138	0.021
1	54	0.161
2	15	0.685
3	4	0.971
4	1	0.997

**Fig. 2** Test software functional flow diagram of stochastic star identification technique.

## Conclusions

We have proposed and formulated a different methodology to the star identification problem. Preliminary results (based on a simple set of statistics with the Gaussian density function and arbitrary chosen covariance matrix) are very promising. The algorithm also performs very well under a large degree of perturbations on measured positions and magnitudes. Because the identification is achieved using a set of statistics formed by the neighboring stars, a relatively large number of stars ( $10^4$ ) in the camera FOV are required. This can be achieved by looking at dimmer stars or increasing the camera FOV. For an application with a wide FOV star camera (20 deg or more), a simple triangle matching technique on observed bright stars may prove superior in terms of performance and implementation simplicity to what has been proposed in this paper. However, the stochastic star identification technique possesses a desired characteristic of not having the required memory and computation grow in a combinatorial manner as the required number of catalog stars increases. Although many algorithms may struggle with a large number of stars detected in the observation field, this approach prefers a high density of stars for better results. Furthermore, because identification is based on the statistics of the observed star group, significantly large perturbations on the measured positions and magnitudes can be tolerated.

For deep-space exploration with small spacecraft, the science imaging camera (0.5 deg–3 deg FOV) may be desired to perform star identification and tracking functions to minimize number of instruments, mass, and power. Space telescopes and interferometers may also desire a fine guidance sensor with extremely narrow FOV (less than 1 deg). The proposed stochastic star identification may prove invaluable for such applications. Continuing research to ad-

dress quantitative performance of various star identification algorithms is in progress.

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